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Availability Modelling of a Wheel Manufacturing Plant (Using Recurssive Differential Equations Method)

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Abstract: To study the availability of complex and complicated repairable systems having two or more states as reduced, working, and failed it is important to fully characterize the states in which a system may be such that the described system regulates some stochastic process. Five smaller units make up a wheel plant. The wheel manufacturing facility's reliability and availability are represented in this study using a time-homogeneous Markov process with diminishing states. It has been found to be a successful approach that is entirely dependent on modeling and numerical analysis.

Keywords: Markov Process, Wheel Plant, Availability, Reliability

1. Introduction

The states of repairable systems with two or more states (reduced, functional, and failed) must be appropriately represented so that the system follows a stochastic process in order to investigate performance metrics for such systems. In this work, the dependability and availability of a wheel manufacturing facility with decreased states are represented using the time-homogeneous Markov process. It is a potent method based solely on modeling and discrete analysis. The system analyst must choose components while analyzing a system with a variety of features in order to sustain fault-free operation. In this research paper, various steady-state parameters of system are investigated, and solved/optimized using the recursive method. The purpose of the paper analysis is to effectively determine reliability function and steady state availability of the operational system process as a Markov process, as well as to get best system designing components which enables long-term, error-free operation, which is necessary for maximum system productivity. This study examines the availability and reliability of a wheel manufacturing plant in order to highlight the importance of the Markov process. A state transition diagram is used to describe the system states, and when the diagram is updated, the reliability function appears. While the normalizing condition is in place, a genetic algorithm is employed to obtain and optimize the expression for steady-state availability. Using a Markov diagram of a wheel manufacturing facility, this study aims to (1) extract a reliability function and an availability function, and (2) formulate an optimization model for steady state availability. The ideal design parameters and system performance indicators are described and accomplished using an exemplary case. The behavior of a bread plant was examined by Kumar et al. in [2018]. In order to do a sensitivity analysis on a cold standby framework made up of two identical units with server failure and prioritized for preventative maintenance, Kumar et al. [2017] used RPGT. Two halves make up the current paper, one of which is in use and the other of which is in cold standby mode. The good and fully failed modes are the only differences between online and cold standby equipment. A case study of an EAEP manufacturing facility was examined by Rajbala et al. [2018] in their work on system modeling and analysis in 2018. A study of the urea fertilizer industry's behavior was conducted by Kumar et al. [2017]. The mathematical formulation and profit function of an edible oil refinery facility were investigated by Kumar et al. in 2017. In a paper mill washing unit, Kumar et al. [2018] investigated mathematical formulation and behavior study. Using RPGT, looked at the Reverse Osmosis Water Treatment Plant. In their study, Kumar et al. [2018] investigated a 3:4:: outstanding system plant's sensitivity analysis. PSO was used by to research limited situations. Using a heuristic approach, Rajbala et al. [2018] investigated the redundancy allocation problem in the cylinder manufacturing plant. Kumar and Rajbal [2018] studied on the reliability and availability analysis using RPGT-A general approach. Kumar [2018] studied on the Performance analysis of a Rice Plant using RPGT.

2. System Description:

This article discusses the wheel manufacturing industry, which satisfies many of our wants and requirements on a daily basis. The subsystems of the wheel manufacturing facility include the gravity die casting machine, gate cutting machine, heat treatment machine, and turning machine. Each subsystem in this plant serves a specific purpose in the overall system's operation because each is essential to the plant's success. The subsystems are all linked together in sequence. The heat treatment machine, a subsystem of the wheel production facility, is detailed in this paper. The operation or failure of any one of the system's components has some effect on how the system functions.

2.1 Consideration and Notations:

- Continuous random variables that are stochastically independent are repair time and failure-free time.
- When fixing a broken system, failures are not considered.
- On a first-in, first-out basis, repairs are made, and a repaired item is regarded as equivalent to a brand-new one.
- P, Q, R, S operational conditions of each of the four main feeding components.
- P,q, r, s failed states of units P, Q, and reduced states of units R, S, respectively.

2.2 State Transition Diagram:

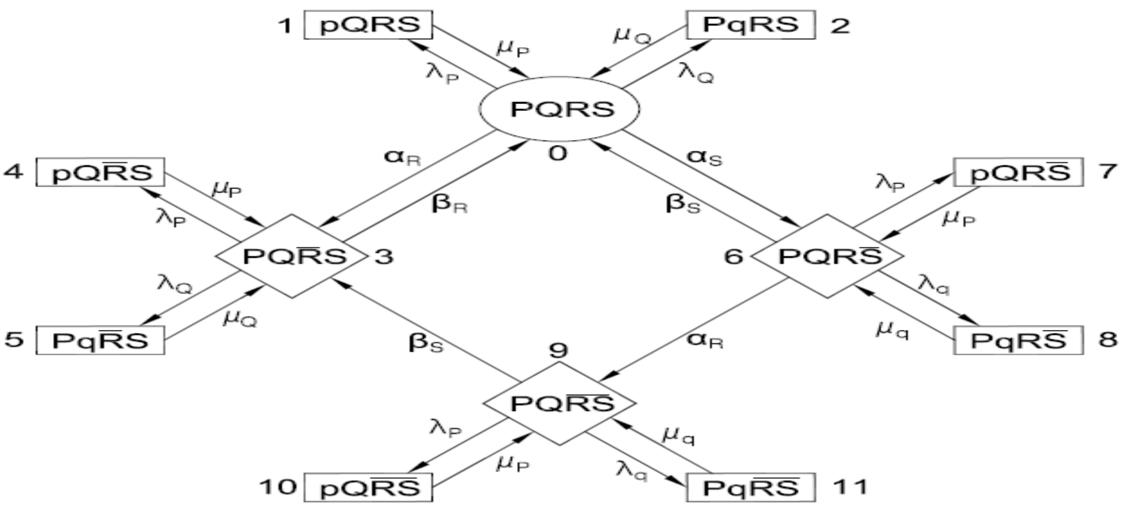


Figure 1: State Transition Diagram

2.3 Model Description:

At first the system in the figure 1 is in fully working when all the units P, Q, R, and S are in fully working state. The failure rates of units P to p are λ_p, Q to q is λ_q. As the unit 'R and S' have sub-components so it works in reduced state. Here $\lambda_p, \lambda_q, \alpha_{r,}$ and α_s are the failure rates of the subsystem. $\mu_p, \mu_q, \beta_{r,}$ and β_s are the repair rates of the subsystem.

2.4 Mathematical Modeling

At any time t, system under study is in any specific state, and it thereafter moves through failed and decreasing states. First order differential equations and the system's state transition diagram are used to create the model's mathematical structure. The following differential equations are connected to the process at the end of this chapter and are the outcome of various probability considerations. The availability expressions for time-dependent and steady-state of the system are defined by solving these equations. The pertinent transition diagram of the system is drawn in Figure 1. To study the system Markov process and governing differential equations can be simply generated from a transition diagram.

$$\frac{dp_0(t)}{dt} + (\lambda_P + \lambda_Q + \alpha_R + \alpha_s)\, P_0(t) = \mu_P\, P_1(t) + \mu_Q\, P_2(t) + \beta_R\, P_3(t) + \beta_s\, P_4(t) \ldots \quad (1)$$

$$\frac{dp_1(t)}{dt} + \mu_P\, P_1(t) = \lambda_P\, P_0(t) \ldots. (2)$$
$$\frac{dp_2(t)}{dt} + \mu_Q\, P_2(t) = \lambda_Q\, P_0(t) \ldots. (3)$$
$$\frac{dp_3(t)}{dt} + (\lambda_P + \lambda_Q + \beta_R)\, P_3(t) = \alpha_R\, P_0(t) + \mu_Q\, P_4(t) + \mu_Q\, P_5(t) + \beta_s\, P_9(t) \ldots (4)$$
$$\frac{dp_4(t)}{dt} + \mu_P\, P_4(t) = \lambda_P\, P_3(t) \ldots (5)$$
$$\frac{dp_5(t)}{dt} + \mu_Q\, P_5(t) = \lambda_Q\, P_3(t) \ldots. (6)$$
$$\frac{dp_6(t)}{dt} + (\lambda_P + \lambda_Q + \beta_s + \alpha_R)\, P_6(t) = \alpha_s\, P_0(t) + \mu_P\, P_7(t) + \mu_Q\, P_8(t) + \alpha_R\, P_9(t) \ldots (7)$$
$$\frac{dp_7(t)}{dt} + \mu_P\, P_7(t) = \lambda_P\, P_6(t) \ldots (8)$$
$$\frac{dp_8(t)}{dt} + \mu_Q\, P_8(t) = \lambda_Q\, P_6(t) \ldots (9)$$
$$\frac{dp_9(t)}{dt} + (\lambda_P + \lambda_Q + \beta_s)\, P_9(t) = \mu_P\, P_{10}(t) + \mu_Q\, P_{11}(t) + \alpha_R\, P_6(t) \ldots (10)$$
$$\frac{dp_{10}(t)}{dt} + \mu_P\, P_{10}(t) = \lambda_P\, P_9(t) \ldots (11)$$
$$\frac{dp_{11}(t)}{dt} + \mu_Q\, P_{11}(t) = \lambda_Q\, P_9(t) \ldots (12)$$

A Markov process is said to be stationary if state probabilities are stable with respect to time [1] and are attained taking the conditions: as $t \to \infty, P_i(t) \to P_i$ and $\frac{dp_i(t)}{dt} \to 0, \forall\, i$.

$$(\lambda_P + \lambda_Q + \alpha_R + \alpha_s)\, P_0 = \mu_P\, P_1 + \mu_Q\, P_2 + \beta_R\, P_3 + \beta_s\, P_4 \ldots. \quad (13)$$
$$\mu_P\, P_1 = \lambda_P\, P_0 \ldots. \quad (14)$$
$$\mu_Q\, P_2 = \lambda_Q\, P_0 \ldots. \quad (15)$$
$$(\lambda_P + \lambda_Q + \beta_R)\, P_3 = \alpha_R\, P_0 + \mu_Q\, P_4 + \mu_Q\, P_5 + \beta_s\, P_9 \ldots \quad (16)$$
$$\mu_P\, P_4 = \lambda_P\, P_3 \ldots \quad (17)$$
$$\mu_Q\, P_5 = \lambda_Q\, P_3 \ldots. \quad (18)$$
$$(\lambda_P + \lambda_Q + \beta_s + \alpha_R)\, P_6 = \alpha_s\, P_0 + \mu_P\, P_7 + \mu_Q\, P_8 + \alpha_R\, P_9 \ldots (19)$$
$$\mu_P\, P_7 = \lambda_P\, P_6 \ldots \quad (20)$$
$$\mu_Q\, P_8 = \lambda_Q\, P_6 \ldots \quad (22)$$
$$(\lambda_P + \lambda_Q + \beta_s)\, P_9 = \mu_P\, P_{10} + \mu_Q\, P_{11} + \alpha_R\, P_6 \ldots (23)$$
$$\mu_P\, P_{10} = \lambda_P\, P_9 \ldots \quad (24)$$
$$\mu_Q\, P_{11} = \lambda_Q\, P_9 \ldots \quad (25)$$

Solving these linear differential equations recursively using normalizing condition, i.e., sum of all probabilities is one in terms of P_0, we have various steady state probabilities as given in below table 1:

Table 1: Various Steady state Probabilities

Now, using the

$P_1 = \frac{\lambda_P}{\mu_P} P_0$	$P_2 = \frac{\lambda_Q}{\mu_Q} P_0$
$P_3 = P_0\alpha/\beta$	$P_4 = \frac{\lambda_P \alpha_R\, \alpha}{\mu_P \beta_R \beta} P_0$
$P_5 = \frac{\lambda_P \alpha_{R\alpha})}{\mu_P \beta_R \beta} P_0$	$P_6 = \frac{\alpha_S}{\beta} P_0$
$P_7 = \frac{\lambda_P \alpha_S}{\mu_P \beta} P_0$	$P_8 = \frac{\lambda_Q \alpha_S}{\mu_Q \beta} P_0$
$P_9 = \frac{\alpha_R \alpha_S}{\beta_S \beta} P_0$	$P_{10} = \frac{\lambda_P \alpha_S\, \alpha_R}{\mu_P\, \beta_S \beta} P_0$
$P_{11} = \frac{\lambda_Q \alpha_S\, \alpha_R}{\lambda_Q \beta_S \beta}$	

normalizing condition $\sum_{i=0}^{11} P_i = 1$ … (27)

$$P_0 = [1 + \frac{\lambda_P}{\mu_P} + \frac{\lambda_Q}{\mu_Q} + \frac{\alpha_R\, \alpha}{\beta_R \beta} + \frac{\lambda_P \alpha_R\, \alpha}{\mu_P \beta_R \beta} + \frac{\lambda_P \alpha_R\, \alpha}{\mu_P \beta_{R\beta}} + \frac{\alpha_S}{\beta} + \frac{\lambda_P \alpha_S}{\mu_P \beta} + \frac{\lambda_Q \alpha_S}{\mu_Q \beta} + \frac{\alpha_R \alpha_S}{\beta_S \beta} + \frac{\lambda_P \alpha_S\, \alpha_R}{\mu_P\, \beta_{S\beta}} + \frac{\lambda_Q \alpha_S\, \alpha_R}{\lambda_Q \beta_S \beta}]^{-1} \ldots (28)$$

Where $\alpha = \alpha_R + \alpha_S + \beta_{S\,and\,,}\beta = (\alpha_R + \beta_S)$

The steady state availability [1] is given by the equation as follows:

$Av_s = P_0 + P_3 + P_6 + P_9$, i.e., sum of the probabilities of all working states

$$= \left[1 + \frac{\alpha_R\, \alpha}{\beta_R \beta} + \frac{\alpha_S}{\beta} + \frac{\alpha_R \alpha_S}{\beta_{S\beta}}\right] P_0 \;\ldots. (29)$$

This expression is reduced using [28],

$$Av_s = \frac{1}{(1+(\lambda_P/\,\mu_P)+(\lambda_Q/\,\mu_Q))} \ldots. \qquad (30)$$

While calculating the dependability function for this Markov process all the down states in the transition diagram are considered as absorbing states. The differential equations associated with the transition diagram produces is as follows:

$$\frac{dp_0(t)}{dt} + (\lambda_P + \lambda_Q + \alpha_R + \alpha_s)\, P_0(t) = \beta_R\, P_3(t) + \beta_s\, P_6(t) \ldots \quad (31)$$

$$\frac{dp_1(t)}{dt} = \lambda_P\, P_0(t) \;\ldots. (32)$$

$$\frac{dp_2(t)}{dt} = \lambda_Q\, P_0(t) \;\ldots. (33)$$

$$\frac{dp_3(t)}{dt} + (\lambda_P + \lambda_Q + \beta_R)\, P_3(t) = \alpha_R\, P_0(t) + \beta_s\, P_9(t) \ldots (34)$$

$$\frac{dp_4(t)}{dt} = \lambda_P\, P_3(t) \ldots (35)$$

$$\frac{dp_5(t)}{dt} = \lambda_Q\, P_3(t) \;\ldots. (36)$$

$$\frac{dp_6(t)}{dt} + (\lambda_P + \lambda_Q + \beta_s + \alpha_R)\, P_6(t) = \alpha_s\, P_0(t) \ldots (37)$$

$\frac{dp_7(t)}{dt} = \lambda_P P_6(t) \ldots$ (38)

$\frac{dp_8(t)}{dt} = \lambda_Q P_6(t) \ldots$ (39)

$\frac{dp_9(t)}{dt} + (\lambda_P + \lambda_Q + \beta_s) P_9(t) = \alpha_R P_6(t) \ldots$ (40)

$\frac{dp_{10}(t)}{dt} = \lambda_P P_9(t) \ldots$ (41)

$\frac{dp_{11}(t)}{dt} = \lambda_Q P_9(t) \ldots$ (42)

Which are linear differential equations with constant coefficients that can be numerically resolved in a variety of ways, in this demonstration, the solution of equations is obtained using the Eigen value technique, with e_i serving as the Eigen values and Vi serving as the corresponding eigenvectors.

$V(t) = \sum_{i=0}^{11} k_i \, exp\,(e_i * t) v_i$ with $V(0) = [1\ 0\ 0 \ldots 0]^T$,

$V(t) = [\,P_0(t) P_1(t) \ldots \ P_{11}(t)]^T \ldots$ (43)

The reliability of system at anytime t is given by $R(t) = P_0(t) + P_3(t) + P_6(t) + P_9\,(t)$. Thus, the reliability & availability of the system containing the Markov process may be easily determined by modeling the corresponding reliability function and availability function transition diagrams.

3. Results and Discussion

The findings of the system's time-dependent availability are computed using MATLAB software. The findings are presented as a table [2] and graph [2].

Table 2: Effect of availability w.r.t. time

Time (months)	Availability
0	1
15	0.95
30	0.91
45	0.84
60	0.80
75	0.73
90	0.69
105	0.61
120	0.50
135	0.46
150	0.40
165	0.36
180	0.28
195	0.22
210	0.15

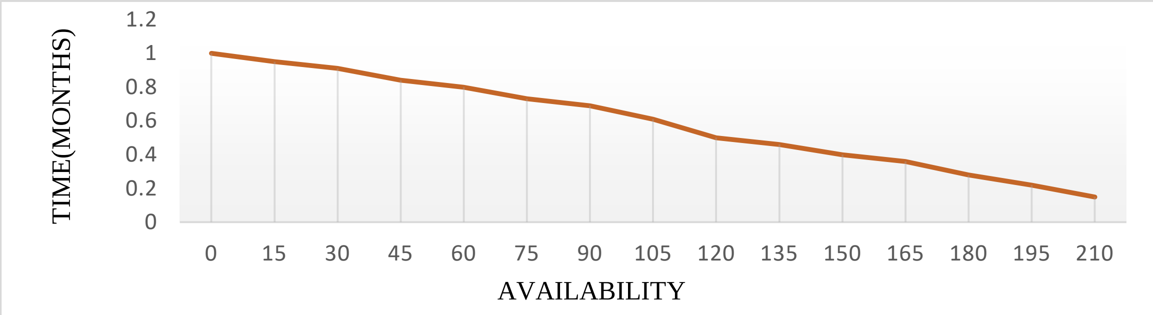


Figure 2: Variation of availability w.r.t. time

3.1 Reliability of the system

Integrating analysis, assurance, and reliability optimization across the design, production, and usage lifecycles is necessary to meet the demands of high product dependability and long life. The reliability of the finished product is closely tied to the effectiveness of the production process and the dependability of the manufacturing system. Even the best design is frequently insufficient to finish the job. The main technique for determining and enhancing the competency of complex systems is the dependability analysis of essential components. The following table [3] and graph [3] present the findings of the reliability analysis:

Table 3: Effect of reliability w.r.t. time

Time (hrs.)	Reliability R(t)
0	1
20	0.934657
60	0.80007
100	0.797067
140	0.654371
180	0.546987
220	0.434201
260	0.354267
280	0.245389
320	0.134526
350	0.045984

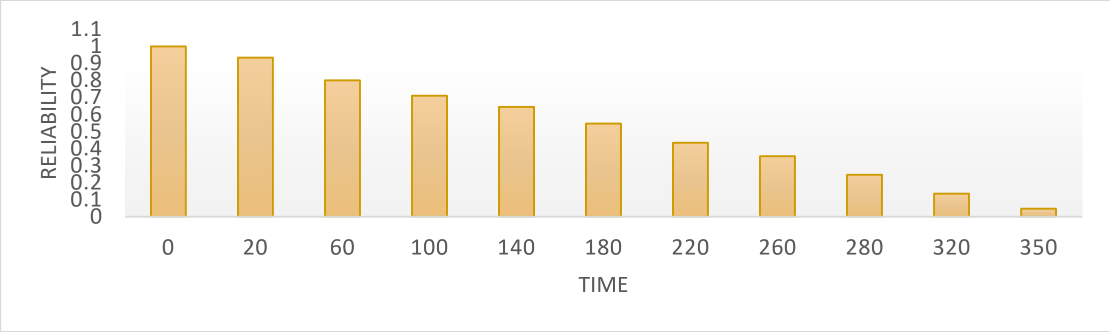


Fig. 3: Variation of reliability w.r.t. time

3.2 Steady-state Availability of the system

The results of steady-state availability are described in form of tables [4] and graphs [4]. The table 4 is developed by fixed the ($\lambda_Q = 0.02$, $\alpha_R = 0.05$, $\alpha_S = 0.07$, $\mu_Q = 0.002$, $\beta_R = 0.005$, $\beta_S = 0.007$) as a constant and varying the value of (λ_P, μ_P).

Table 4: Effect of failure and repair rates of GDC machine on availability

$\lambda_P \rightarrow$ $\mu_P \downarrow$	0.01	0.03	0.05	0.08	0.11	Fixed Value
0.001	0.7581	0.7215	0.7136	0.7078	0.7003	$\lambda_Q = 0.02$, $\alpha_R = 0.05$, $\alpha_S = 0.07$, $\mu_Q = 0.002$, $\beta_R = 0.005$, $\beta_S = 0.007$
0.003	0.8170	0.7580	0.7350	0.7240	0.7072	
0.005	0.8860	0.7920	0.7570	0.7410	0.7311	
0.008	0.9270	0.8205	0.7816	0.7506	0.7148	
0.011	0.9430	0.8621	0.8040	0.7750	0.7550	

The availability values for different values of failure and repair rates are shown in Table No. 4. The availability values range from 0.7003 to 0.9430. The values of the table [4] are represented as a graph [4], as follows:

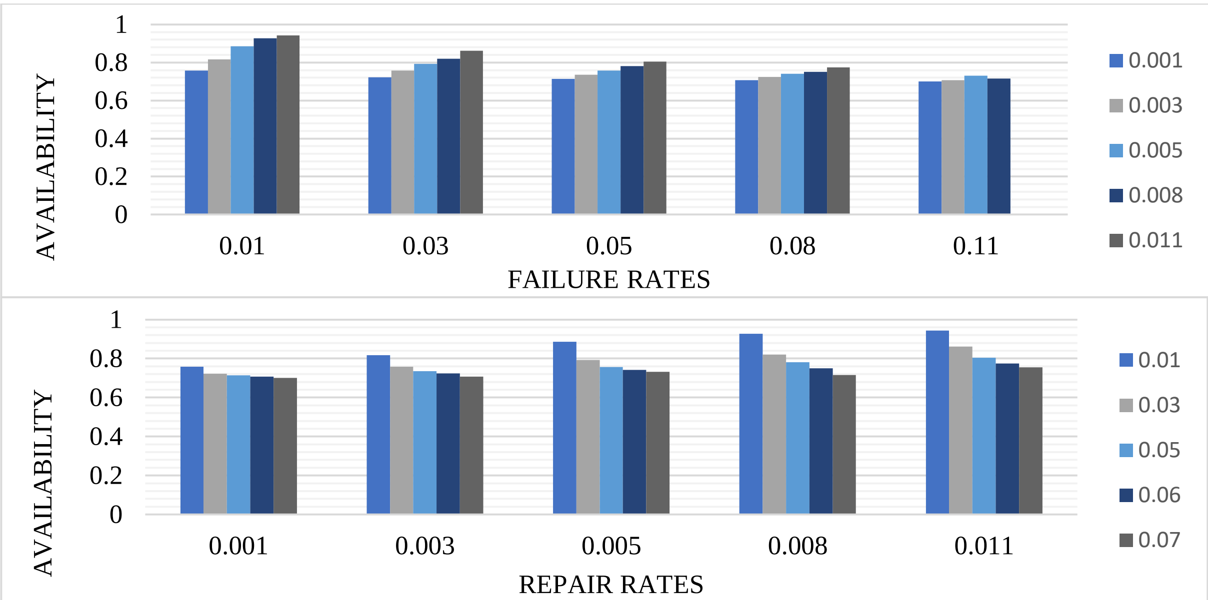


Fig. 4: Variation of availability w.r.t. failure and repair rates of GDC machine

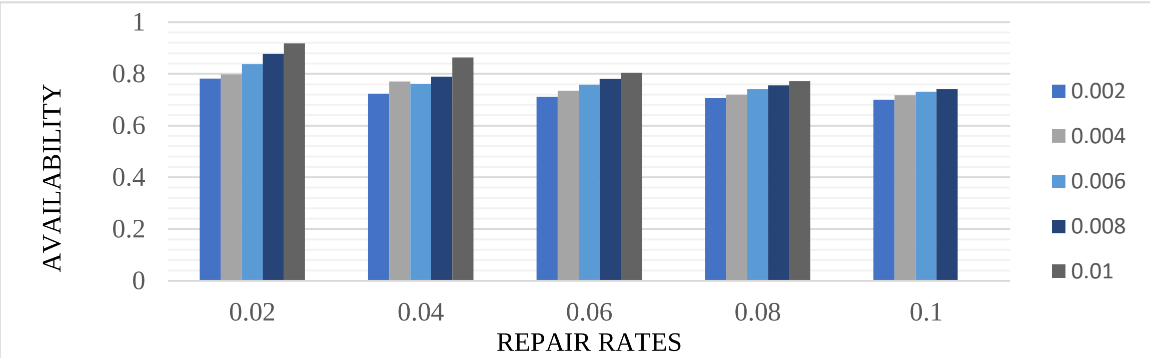
The importance of the system's availability reduces as failure rates rise.

The table 5 is developed by fixed the (λ_P = 0.01, α_R= 0.05, α_S = 0.07, μ_P= 0.001, β_R= 0.005, β_S = 0.007) as a constant and varying the value of (λ_Q, μ_Q).

Table 5: Effect of failure and repair rates on availability of Gate Cut Machine

$\lambda_Q \rightarrow \mu_Q$ ↓	0.02	0.04	0.06	0.08	0.10	Fixed Value
0.002	0.7816	0.7240	0.7106	0.7060	0.7001	λ_P = 0.01, α_R= 0.05, α_S = 0.07, μ_P= 0.001, β_R=0.005, β_S = 0.007.
0.004	0.7981	0.7708	0.7350	0.7201	0.7170	
0.006	0.8370	0.7605	0.7580	0.7403	0.7311	
0.008	0.8767	0.7890	0.7811	0.7560	0.7408	
0.010	0.9182	0.8640	0.8040	0.7718	0.7520	

Table No. 5 shows the different failure and repair rates. The values of availability vary from 0.7001 to 0.9182. The values of table [5] are expressed in form of graph [5] as shown below:



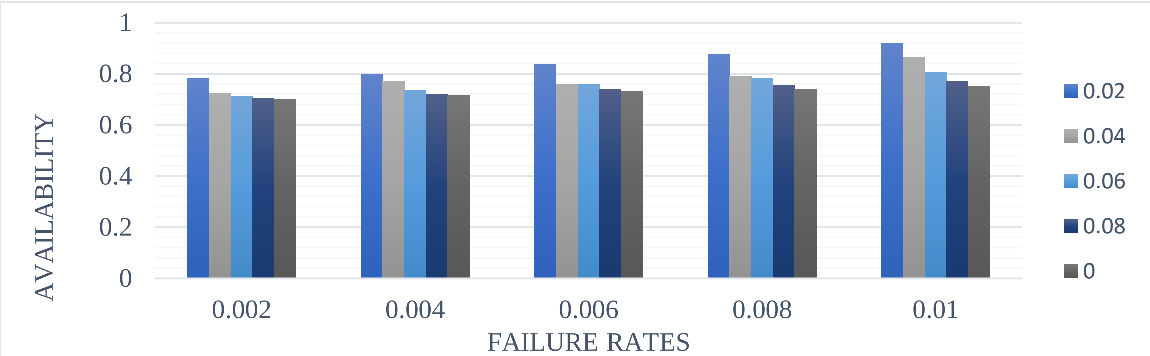


Fig. 5: Variation of availability w.r.t. failure and repair rates of Gate Cut Machine

4. Conclusion

It is obvious that the GDC Machine subsystem requires the most maintenance. Because of its significantly higher repair rate on availability compared to other subsystems, subsystem GDC should be given priority. Table 2 indicates that the system's time-dependent availability decreases over time. Table 3 shows that as the period goes on, the system's reliability declines. The maximum value of availability from table 4 is 0.9430, which is attained when the failure rate is the lowest (0.01) and the repair rate is the highest (0.011). The maximum availability is 0.9182, according to table 5. The best availability, according to the aforementioned findings, is 0.9430.

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